Potty Parity: Process Flexibility via Unisex Restroom

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ABSTRACT

We study the problem of inequitable access to public restrooms by women and the LGBTQ+ community. Individuals enter a restroom based on their gender identity and the expected (or observed) wait time. We consider two measures of potty parity: first, the conventional wait-time parity, and second, our proposed utility parity, which encompasses both wait time and gender identity to estimate users' utility for using a restroom. We show the benefits of unisex restrooms analytically and from various angles: (a) reducing the wait time for the women's restroom; (b) enhancing the potty parity of wait times and users' utility; (c) increasing users' feelings of safety; and (d) shrinking the wait-time disparity when arrival rates fluctuate. Moreover, we provide insights into both renovating existing buildings and designing restrooms from scratch. In particular, we show the following: (i) The process flexibility of having a one-unit unisex restroom, either by converting a unit of the men's restroom or building an additional one, goes a long way toward improving wait time or user utility, and reducing their disparities. (ii) Building the women's room and the unisex restroom next to each other (such that users can jockey lines) improves potty parity. (iii) Even though an all-unisex restroom leads to parity of wait times, surprisingly, it does not improve utility potty parity, but reverses the ranking of users' utility in the population. (iv) Providing an all-unisex room plus urinal(s) can increase efficiency still more.

KEYWORDS

potty parity, inclusion, process flexibility, queueing

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1 INTRODUCTION

Providing fair access to public restrooms has been an issue for so long in our society that the term *potty parity* was coined to describe this specific issue. Potty parity is defined as "equitable access of all users to public restrooms." There are at least two main problems with the current design of public restrooms that the media highlight in the news. First, "for a typical busy bathroom, women have to wait about 34 times as long as men. But, contrary to some stereotypes, it's not because they're busy chitchatting or reapplying their makeup inside."[1] Second, in a podcast, Sandy Allen from upstate New York, speaking as a non-binary and transgender person whose gender expression lands between "female" or "male," said, "Some strangers call me sir, others call me ma'am. So public restrooms always present this dilemma. In women's rooms, people look at me weird or say mean things. But I am often too afraid to go into men's rooms. I am sure, and I have this voice, but I am too worried that those straights could make me a target. It can feel risky and exhausting to navigate these two bad options." Sandy continued by saying, "I am in the New York City subway, and I did not find a bathroom earlier, and it has been hours, and that is difficult, but I am just going to hold it for more hours."[2]

The first major problem is that women tend to spend much more time waiting in queues to use public restrooms than men.[3] This is the case especially in older buildings that used to be male-dominated environments, due to the disproportionate allocation of the number of plumbing fixtures (hereafter, "fixtures") to women's and men's restrooms. The International Plumbing Code (IPC) that sets the quota for the minimum number of gender-segregated restrooms a building has to provide acknowledged the disparity of wait times between the women's and men's restrooms and required equal square footage for the two restrooms in 2004. However, this requirement did not resolve the problem because women's restrooms occupy more space (urinals require less space than individual toilet stalls), and thus, the number of fixtures in a women's restroom will be less than the number in a men's of equal size. IPC 2009 revised the requirement, stipulating instead an equal number of fixtures. However, this requirement still could not resolve the issue because women tend to spend more time in restrooms due to physiological differences, and often accompany an elderly woman or a child to the washroom. IPC 2015 revised its requirement to a 2-to-1 ratio of fixtures in favor of women. In a numerical study, Huh et al. [7] show that even this ratio does not fully resolve the issue. In view of the IPC requirement from 2004 to 2015, we can assume that buildings built and renovated before 2015 would suffer from disparity of wait times for the women's and men's restrooms.

The second major problem is that trans and gender non-conforming people do not have access to safe and comfortable public restrooms. Gender-segregated restrooms tend to be an anxiety-provoking environment for individuals whose gender expression is neither male nor female, like Sandy. They may encounter an unwelcome climate or even verbal or physical assault when entering either of these restrooms.[4] Lack of access to public restrooms leads to increased physical and mental problems among the LGBTQ+ community; see Price-Feeney et al. [10]. There is an increase in demand

[1] https://www.insider.com/why-women-always-wait-longer-bathroom-public-restroom-2019-9

[2] https://99percentinvisible.org/episode/where-do-we-go-from-here/

[3] https://www.theatlantic.com/family/archive/2019/01/women-men-bathroom-lines-wait/580993/

[4] https://www.rollingstone.com/culture/culture-news/what-its-like-to-use-a-public-bathroom-while-trans-65793/

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for unisex/all-gender/gender-neutral restrooms, with people signing petitions and submitting requests to facilities for these types of restrooms.[5] However, many buildings have still not adopted unisex restrooms, or the number and location of the unisex rooms provided are not sufficient to give equal access to all users.

In view of these two major problems, we study whether and to what extent adopting unisex restrooms helps with both the long wait time for the women's restroom and the lack of access to a restroom aligned with one's gender identity. One challenge is how to measure potty parity. Looking into the literature (see, e.g., Huh et al. 7) and practice (see, e.g., IPC 2004-2021), we find that the efforts have been directed at equating wait times for the women's and men's restrooms, which we refer to as *wait-time parity*. Even though this effort may address the first problem, it does not solve the second. We propose a more inclusive measure, *utility parity*, which accounts for users' gender identity in addition to their wait time. This measure seeks a design for all users that reduces the gap between the utilities of any two arbitrary users using the service. Therefore, this measure of utility parity is people-centric and tries to consider every individual and reduce the disparity of their utilities from the service.

Our work is related to three research streams. First, we draw inspiration from the queueing literature, in particular those papers that use queueing methodology to study *socioeconomic* behavior in service systems; see [1], [14] , and [4]. In addition, the way we model how people choose which restroom to go is consistent with the queueing economics literature, in which rational customers choose the line that gives them the highest utility as a function of quality and wait time (which depends on service capacity) under a given (observable, unobservable, or another) information structure of the system, see [13] and Yang et al. [15].

Second, our paper contributes to the literature on process flexibility. Jordan and Graves [8] study the benefits of a little process flexibility in production plants. Since then, many papers have explored and demonstrated the benefit of having a bit of process flexibility in various operational settings; see Gurumurthi and Benjaafar [5], Bassamboo et al. [2]. We show that having a little restroom flexibility through unisex rooms contributes significantly to potty parity. In contrast, Sunar et al. [11] show when delay-sensitive customers can decide to join or balk in an observable queueing system, pooling queues can result in strictly smaller social welfare than the dedicated system. Our work shows the advantage of dedicated services when a portion of users spend less time using a dedicated server in an unobservable queueing setting. In particular, we show that having dedicated stalls for cis and trans men who use the restroom to urinate can increase system efficiency compared to an all-unisex restroom.

Third, our research is related to the literature on gender issues in operations management (see Cachon et al. 3 for references of papers on ethics, equity, and well-being in operations management). [9] study the pricing (uniform pricing vs. price discrimination) and capacity allocation for two classes of customers when customers' utility depends on how many and who the other class of customers using the service are. They use the motivating example of a bar with two classes of customer—women and men, and thus, customers' utility depends on the price they pay as well as the portion of each gender using the bar. [12] consider ride-hailing platforms where female riders and drivers may have safety concerns about being matched with male drivers and riders respectively. In particular, the authors study the design of a gender-specific service for ride-hailing systems. In our work, we also consider the safety concerns of the users motivated by reports on the verbal and sexual harassment of transgender people in public restrooms. We show that unisex restrooms can reduce safety concerns for all users. Huh et al. [7] is the paper closest to ours, as they also study the potty parity problem. Using simulation, they show that to bridge the potty parity gap between the women's and men's rooms, the women's room requires at least twice as many fixtures as the men's room. In contrast, we analyze the potty parity problem by formulating the utility function of the users depending on their gender identity and wait time. We include non-binary users, making it possible to consider the effect of service design on all users without classifying or labeling individuals based on their gender. Using the utility model, we focus on the benefits of the unisex restroom in reducing the disparity in wait times as well as the disparity of utilities among users.

2 MODEL

Now we provide a utility model for users of public restrooms on entering gender-segregated and unisex rooms. We consider users' utility to consist of the reward they gain on using a restroom minus the disutility of waiting in line and the disutility associated with the mismatch between some users' gender identity and the restrooms' signage. We next use the queueing model to formulate users' disutility from waiting in line, and then we use the Hotelling model to formulate the disutility associated with users' gender identity. We assume users are utility maximizers, and thus, when we study and compare the restrooms' wait times and the users' welfare in different restroom layouts, this model helps predict the portion of the users entering each of the restrooms.

Wait-time disutility for gender-segregated rooms. We consider gender-segregated rooms with signage for women and men. Restroom legislation in some countries and states requires individuals to use public gender-segregated restrooms in accordance with their birth-certificate identity, while the legislation in others requires them to use public gender-segregated restrooms in accordance with their gender identity. Therefore, a trans woman may need to use the men's room in Tennessee, while she can use the women's room in Massachusetts. This study assumes that all users will comply with the facilities' restroom bill and choose which gender-segregated restroom to enter accordingly. Based on the policy of the facility/state, when there only exist gender-segregated restrooms, let $\mathcal{W}$ denote the set of users who enter the women's room with a given arrival rate Λ_0, and $\mathcal{M}$ denote the set of users who enter the men's room with a given arrival rate Λ_1.

We denote by S_0 and S_1 the service-time distributions of a user $i \in \mathcal{W}$ and $j \in \mathcal{M}$, respectively. We model the women's room as an M/G/n_0 queue, where n_0 denotes the number of fixtures in the women's room, and use $W_0(\lambda_0, n_0)$ to denote the expected wait time for the women's restroom, where λ_0 is the effective Poisson

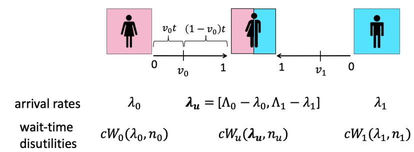
[5] https://seattlespectator.com/2019/10/17/students-petition-law-school-to-de-gender-bathrooms/

arrival rate of the users there. We model the men's room as an M/G/n_1 queue, where n_1 denotes the number of fixtures in the men's room, and use $W_1(\lambda_1, n_1)$ to denote the expected wait time for the men's restroom, where λ_1 is the effective Poisson arrival rate of the users there. [6] When the stability condition $\lambda_l < \mathbb{E}[S_l]$ for $l = \{0,1\}$ is satisfied, the expected wait time $W_l(\lambda_l, n_l)$, $l = \{0,1\}$ is a non-negative number, though our analysis does not require $W_l(\lambda_l, n_l)$ to be finite per se. Our analysis does not rely on a specific form of a queueing formula for the steady-state expected wait time. Instead, it only uses some intuitive properties of the expected wait time of a system with respect to its parameters. We denote by $c > 0$ the (homogeneous) waiting cost coefficient per unit of time waiting in a line, and thus, the users of the women's room incur waiting cost $cW_0(\lambda_0, n_0)$ standing in line and the users of the men's room incur waiting cost $cW_1(\lambda_1, n_1)$ standing in line. For ease of exposition, we consider linear wait costs with the same wait-time coefficient for all users of both restrooms. However, our results are robust for general wait-cost functions, which can be heterogeneous for $\mathcal{W}$ and $\mathcal{M}$.

Wait-time disutility for unisex rooms. We consider non-segregated restrooms, which can take the form of multi-fixture restroom (i.e., multiple rooms with shared sinks) or multiple single-occupancy rooms (i.e., each room has its own sink) with unisex signage, or family rooms. We refer to this as a *unisex* room, which users of all genders can use. When a unisex room is available, users can decide whether they want to use the gender-segregated restrooms in the way consistent with the facility's policy, or the unisex room. We assume users do not have an outside option; for instance, consider students at school who need to use the restroom during recess or football fans in a stadium who need to use the restrooms during halftime. In this case, users stay in line even though they may not feel comfortable, and the queue may be long. We model the unisex restroom as a queueing system where we denote by $\boldsymbol{\lambda_u} = [\Lambda_0 - \lambda_0, \Lambda_1 - \lambda_1]$ the vector of users' effective arrival rates at the unisex room from $\mathcal{W}$ and $\mathcal{M}$, where λ_0 and λ_1 are the effective arrival rates at the women's and the men's rooms, respectively. We denote by $\boldsymbol{S_u} = [S_0, S_1]$ the vector of random service times indicating that users from $\mathcal{W}$ enter the unisex room with rate $\Lambda_0 - \lambda_0$ and with service time drawn from S_0; and users from $\mathcal{M}$ enter the unisex room with rate $\Lambda_1 - \lambda_1$ and with service time drawn from S_1. We denote by $W_u(\boldsymbol{\lambda_u}, n_u)$ the expected wait time for the unisex room, where n_u denotes the number of fixtures in the unisex room. We consider that $W_u(\boldsymbol{\lambda_u}, n_u)$ is a non-negative number when the queue is stable, and our analysis can also handle the case when it is infinity. Hence, users of the unisex restroom incur waiting disutility $cW_u(\boldsymbol{\lambda_u}, n_u)$.

Gender-identity disutility. We capture the heterogeneous customer preferences in a gender-segregated restroom[7] vs. the unisex room by a Hotelling model. More specifically, we consider a Hotelling line between the women's and the unisex room for users $i \in \mathcal{W}$ where the user's location in the line indicates their type,

Figure 1: gender-identity disutility



Note. v_l follows a general probability density distribution $f_l(v)$, $l = 0, 1$. When we have gender-segregated-restroom-only designs, $\lambda_0 = \Lambda_0$, and $\lambda_1 = \Lambda_1$.

which we denote by $v_i \in [0,1]^{\mathcal{W}}$.[8] Users are heterogeneous in terms of their gender identity and their relative comfort level with using a unisex restroom compared with a gender-segregated room, and thus, their type is drawn from a probability distribution function $f_0(v)$. Likewise, we consider a Hotelling line between the men's room and the unisex, where users $j \in \mathcal{M}$ have a type denoted by $v_j \in [0,1]^{\mathcal{M}}$ drawn from a probability distribution function $f_1(v)$. Therefore, a user of type v will incur gender-identity disutility tv on using a gender-segregated restroom and gender-identity disutility $t(1-v)$ on using a unisex restroom. We denote by t the gender-identity sensitivity to using a restroom with specific signages such as women's, men's and unisex rooms in the population. Our model can accommodate heterogeneous sensitivity to gender identity as follows. After we normalize t to a constant, a different sensitivity level can be absorbed into the distribution along the Hotelling line. For example, there may be some users who are indifferent to the choice between the unisex room and the gender-segregated one, i.e., $t = 0$. After the normalization, this group of users will stand in the middle of the Hotelling line for a constant t.

We believe that considering gender-identity disutility in the model is crucial. For example, the American 2019 National School Climate Survey states that at least 45.2% of LGBTQ students feel unsafe and uncomfortable in gender-segregated restrooms at school.[9] As well, a trans woman may feel uncomfortable using the women's room because she may experience an unwelcoming environment, or even physical attacks.[10] Thus, considering such gender-identity disutility in the model pushes us a step forward toward also considering the utility disparity among users. Moreover, we assume that $f_0(v)$ and $f_1(v)$ are continuous functions in v, and their cumulative distribution function (C.D.F.) is strictly increasing in v. This assumption is inclusive to all users over the whole spectrum and avoids wrongfully labeling or categorizing individuals into strict categories such as women, men, trans, non-binary, etc.

Utility model. We focus on restrooms with different signage that are at a distance or are located such that users cannot see

[6] We do not assume the type of fixtures, i.e., *urinals* vs. *toilet stalls*, to keep our model as parsimonious as possible. However, our results hold in these more detailed models.
[7] Note that in our model, we assume that the users of a gender-segregated restroom have already endogenously determined which restroom to use and will not opt to use the other gender's restroom. This implies that those users know that they will not be welcomed in the other gender's restroom, and that the cost of using it is high.

[8] Note that an alternative model considering only one Hotelling line, where the gender-segregated restrooms are at two ends with the unisex restroom located halfway between them, will be mathematically equivalent to our model when we rescale the two Hotelling segments between the gender-segregated and the unisex rooms to [0,1].
[9] https://www.glsen.org/sites/default/files/2021-04/NSCS19-FullReport−032421−Web$_0$.pdf
[10] https://www.rollingstone.com/culture/culture-news/what-its-like-to-use-a-public-bathroom-while-trans-65793/

the queues at other restrooms. In this respect, we consider the expected wait time of an unobservable queue when analyzing the base model. (However, in Section **??** we provide the analysis and implications for the case in which users can see other restrooms' wait lines and therefore can jockey to the room with fewer users in line.) We assume that users will use their past restroom experiences to anticipate the expected wait time/cost of using gender-segregated vs. unisex rooms. For instance, we express the wait cost for users from $\mathcal{W}$ to use the women's room as $cW_0(\lambda_0, n_0)$. Therefore, we assume that users who take into account their past experiences will develop a habit of entering one type of restroom rather than constantly switching over. We express the utility of a user of type v_l who uses the gender-segregated room $l \in \{0, 1\}$ and the unisex room as $U_l(v_l; \lambda_l, n_l) = r - tv_l - cW_l(\lambda_l, n_l)$, $U_u(v_l; \boldsymbol{\lambda_u}, n_u) = r - t(1 - v_l) - cW_u(\boldsymbol{\lambda_u}, n_u)$, where r is the intrinsic reward that one gains from using a restroom. The value of r is not dependent on which restroom one enters, as individuals share the same experience of relief when they use a restroom. However, the value of r can vary across user types and also can be random as users may return to the restroom for different purposes, i.e., urination, defecation, changing a sanitary napkin or tampon, etc. Essentially, in our model, under no balking option, as users select which restroom to enter, the value of r will cancel out and will not affect the users' choice. Therefore, for ease of exposition, we just refer to the reward as a single value r.

Furthermore, Huh et al. [7] focus on the probability of waiting as an index to measure the potty disparity between the women's and the men's room. We complement their studies by taking an alternative measure of time spent standing in line, which can be particularly relevant during school recess or half-time at stadiums, when the restrooms get busy. However, the two measures are related, and we can derive the same results and messages by using either of them.

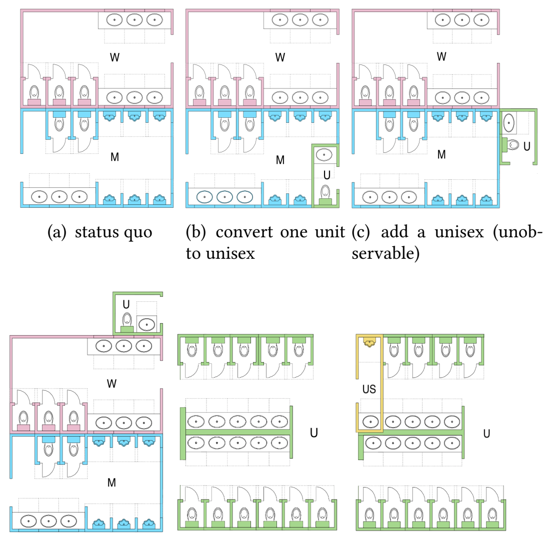
Lastly, note that users may gain a negative utility from using a restroom in our model. As we assume that users do not have an outside option, such as leaving the school or stadium during break times to use another restroom and then return, users will remain in the queue even though they will gain a negative utility from the experience of waiting for too long or entering a restroom in an anxiety-provoking environment.

User equilibrium. In a setting with women's, men's, and unisex rooms, a user equilibrium must be of a threshold type. As user types are continuous, a single user's decision about which room to enter will not affect the rooms' expected wait time. We show the existence and uniqueness of a user equilibrium (see Lemma A.1 in Online Appendix A).

3 RESULTS

Status quo. We start with a status quo in which an existing facility contains only gender-segregated restrooms, with the usual disparity in the desired numbers of fixtures between the women's and men's rooms (see, e.g., Figure 2(a)). We assume that the disparity of wait times between gender-segregated rooms is sufficiently large that even if one fixture is taken from the men's restroom and added to the women's, the women's room will still suffer from longer expected wait times than the men's room. That is, $W_0(\Lambda_0, n_0 + 1) > W_1(\Lambda_1, n_1 - 1)$. This assumption holds for many buildings where gender-segregated restrooms create a huge disparity of wait times between women's and men's rooms. One major contributing factor is that, as mentioned, older versions of the International Plumbing Code (IPC) were inefficient in addressing this disparity, especially for buildings built or renovated before 2015. Moreover, many of the existing buildings have not adopted unisex restrooms. Hence, we assume that many old buildings are far from potty parity and require renovation or retrofitting to install unisex restrooms. For the ease of exposition, we denote by $w_0 := W_0(\Lambda_0, n_0)$, and $w_1 := W_1(\Lambda_1, n_1)$, the expected wait time of the women's and men's rooms in the status quo, respectively; and we denote by $u_0(v_0)$ the utility of a user $v_0 \in [0, 1]^{\mathcal{W}}$, and by $u_1(v_1)$ the utility of a user $v_1 \in [0, 1]^{\mathcal{M}}$ in the status quo.

Figure 2: Example of Restroom Layouts



(a) status quo (b) convert one unit to unisex (c) add a unisex (unobservable) (d) add a unisex (observable) (e) all-unisex (f) all-unisex + urinal stalls

Note. The number of fixtures in the status quo is based on the study of [7], i.e., 3 water closets for the women's restroom (W), and 2 water closets and 6 urinal stalls for the men's restroom (M). We study the adoption of unisex rooms (U) and urinal stalls (US) in different layouts.

Converting One Men's Restroom to Unisex. Suppose an existing facility wishes to provide a unisex restroom but has budget and space constraints. We show that even in this case, converting only *one* unit of the men's room to unisex has at least three benefits (see, e.g., Figure 2(b)).We simplify the notation by using W_0^*, W_1^*, W_u^* to refer to the expected wait time of the women's, the men's, and the unisex room in equilibrium, respectively. We also denote by $U_0^*(v_0)$ and $U_u^*(v_0)$ the equilibrium utility of a user $v_0 \in [0, 1]^{\mathcal{W}}$ entering the women's and the unisex restroom, respectively. Similarly, we denote by $U_1^*(v_1)$ and $U_u^*(v_1)$ the equilibrium utility of a user $v_1 \in [0, 1]^{\mathcal{M}}$ entering the men's and the unisex restroom, respectively. First, it reduces the wait time for the users of the women's room and improves all cis and trans women's utility because of the spillover of the original users of the women's room to the unisex restroom (see Proposition A.1).

Second, this small modification—the conversion of just one unit—reduces the disparity of the wait times between the women's and men's rooms under some conditions stated below.

Proposition 1 (Less Wait Time Disparity). *Suppose that we convert a unit of the men's room to a unisex restroom. In equilibrium, we have at least one of the two following outcomes: (i) The disparity in wait times between the two gender-segregated restrooms will drop, i.e.,* $W_0^* - W_1^* < w_0 - w_1$. *(ii) All users will gain higher utility, i.e.,* $u_0(v_0) < \max\{U_0^*(v_0), U_u^*(v_0)\}$ *for all* $v_0 \in [0,1]^{\mathcal{W}}$*, and* $u_1(v_1) \leq \max\{U_1^*(v_1), U_u^*(v_1)\}$ *for all* $v_1 \in [0,1]^{\mathcal{M}}$.

Third, such a conversion of one unit reduces the utility disparity among the original users of the women's room. Each user would either have less wait time, or would have access to a restroom that is more aligned with their gender identity. Similarly, the utility disparity drops among the original users of the men's room as well.

Theorem 1 (Less Disparity in Utilities within Hotelling Lines). *Suppose that we convert a unit of the men's room to a unisex restroom. The disparity of utilities among users from* $\mathcal{W}$ *will drop. The same is true for users from* $\mathcal{M}$.

Moreover, suppose that we convert $k > 1$ units of the men's restroom to unisex. We denote by $\lambda_0^{(k)}$ and $\lambda_1^{(k)}$ the equilibrium portion of users entering the women's and the men's rooms respectively when k units of the men's rooms are converted to unisex. By convention, let $\lambda_0^{(0)} = \Lambda_0$ and $\lambda_1^{(0)} = \Lambda_1$. Let $\bar{k} > 2$ be the smallest number of the rooms that, if converted to unisex, results in the wait time for the women's restroom becoming less than that for the men's restroom.

Proposition 2 (Marginal Effect of More Unisex Rooms). *Suppose* $W_0(\lambda_0^{(k-1)}, n_0 + k) \geq W_1(\lambda_1^{(k-1)}, n_1 - k)$ *for all* $k \leq \bar{k}$*, where* $\bar{k} > 2$*. The marginal benefit of converting the* k*th unit for* $k < \bar{k}$ *is that either that the disparity of wait times between the two gender-segregated restrooms will drop, or all users will gain higher utility.*

Furthermore, we may say that $\hat{k} := \min_k\{W_0(\lambda_0^{(k-1)}, n_0 + k) \leq W_1(\lambda_1^{(k-1)}, n_1 - k)\}$ is the optimal number of units from the men's restroom that we need to convert to unisex rooms to improve the wait-time disparity or all users' utility.

The takeaway from these results is that the flexibility conferred by even a single-unit converted unisex washroom will go a long way: it will always improve the utility disparity between washroom users and will sometimes reduce the wait-time disparity between gender-segregated restrooms. Moreover, we show that if users' sensitivity to their gender identity is sufficiently high, the wait time for the unisex room will ramp up. This implies that those users who prefer to use the unisex room despite its long wait time must gain greater utility than they do using a gender-segregated restroom, which highlights the importance of considering users' utility in designing public restrooms.

Benefits of an extra unisex room. Suppose the facility is not constrained by budget and space limitations. In this case, building a *new* unisex restroom will bring more benefits than just converting a unit from the men's room to a unisex room (see, Proposition A.2, and for an example of such a layout see Figure 2(c)). However, care must be exercised in the location of the unisex room. We show that adding this new unit close to the women's room (see, e.g., Figure 2(d)) will add extra value.

Proposition 3. *Suppose* $t = 0$*. Adding an extra unisex unit next to the women's room, so that each room is observable to the other, is a Pareto-improving design compared with locating all three rooms in an unobservable system. Moreover, the utility disparity, between any pair of users with one from* $\mathcal{W}$ *and the other from* $\mathcal{M}$*, is less in the observable system compared with the unobservable system.*

The extra value of locating the unisex restroom next to the women's restroom is because users can jump into the other room if it becomes empty, which will reduce the maximum wait-time disparity among the women's, men's and unisex rooms. On the other hand, locating the unisex unit close to the men's room is not desirable. This is because an original user of the men's room, who is already privileged by experiencing shorter wait times, may jump into the unisex room when it is empty and make other unisex-room users who do not have a more suitable alternative wait longer. We show the result for the general case ($t \geq 0$) in Proposition A.3 in Online Appendix A.

Optimal design of an all-gender restroom. Suppose we can design the new facility from scratch. In Proposition 2, we determine the optimal number of women, men and the unisex restrooms given a fixed total number of restrooms. We next study the widely advocated design of an all-unisex room (see, e.g., Figure 2(e)).[11]

Proposition 4 (All-Unisex Restrooms). *We compare the performances of the all-unisex restroom design with that of the status quo (gender-segregated) restroom having the same total number of fixtures. (i) The wait-time disparity among all users is zero in the all-unisex restroom design. (ii) The utility disparity between any two users from the same Hotelling line is the same in both designs, but the ranking of utilities within each Hotelling line is reversed when moving from one design to the other. (iii) If* $\mathcal{E} > 0$*, there exists a threshold* $\hat{t}_1 \geq 0$ *on the sensitivity to gender identity such that for* $t \geq \hat{t}_1$ *(resp.,* $0 \leq t < \hat{t}_1$*), the total social welfare produced by the all-unisex restroom design is higher (resp., lower) than the status quo. (iv) If* $\mathcal{E} < 0$*, there exists a threshold* $\hat{t}_2 \geq 0$ *on the sensitivity to gender identity such that for* $t \geq \hat{t}_2$ *(resp.,* $0 \leq t < \hat{t}_2$*), the total social welfare produced by the all-unisex restroom design is lower (resp., higher) than the status quo.*

Although this design will undoubtedly lead to parity of wait times for all users, contrary to the conventional wisdom, we show that the utility disparity among users will remain the same but with users' utility ranking reversed. In a gender-segregated restroom, users who prefer that arrangement will enjoy higher utility than those who prefer a unisex restroom. However, in an all-unisex restroom, the former users will enjoy lower utility than the latter. We also show that if the group who prefer gender-segregated restrooms is large enough, total welfare will suffer if an all-unisex restroom is built, compared with only gender-segregated rooms. This is because that although building an all-unisex restroom reduces the gender-identity disutility for some users, it ignores the

[11]https://www.buildings.com/articles/27665/all-gender-restrooms-now-comply-code

discomfort of the other users, and hence, this design may not lead to utility parity. Moreover, we propose a Pareto-improving design that surpasses the all-unisex design in terms of wait time and utility, in which there are some toilet stalls for all users and some urinals specific to the original users of the men's room for the purpose of urination only (see, Proposition A.5 and for an example of a layout see Figure 2(f)).

4 EXTENSIONS

4.1 Safety Concerns

The main argument of opponents of unisex public restrooms is about safety, especially for women and children. However, research shows that there is no statistical evidence to support the relationship between discriminatory policies and crimes in washrooms (Hasenbush et al. 6). Even so, we show that one of the benefits of a unisex restroom is to alleviate the safety concerns of users. We define an index that captures the perception of safety for a user.

Define 1 (Safety Concern). *In equilibrium, we refer to a safety concern index SC_l for restroom types $l = 0, 1, u$ by the difference between the two extreme types using the same restroom. That is, $SC_0 = \bar{v}_0$, $SC_1 = \bar{v}_1$, and $SC_u = 2 - \bar{v}_0 - \bar{v}_1$ denote the safety concern index of the women's, men's and unisex rooms, respectively, where $\bar{v}_0$ and $\bar{v}_1$ are the cutoff points on the women's and men's Hotelling lines at which users are indifferent about which alternative they use in equilibrium.*

Definition 1 states that when the range of users with different gender identities entering the same restroom is narrower, users' perception of safety is higher. We capture safety concerns in such a way because cases in which a user encounters an unexpected event matter when it comes to safety. For instance, users in the women's room do not expect to see a man in the restroom. If a user with a male gender expression enters a women's room, some users may feel stressed and unsafe. On the other hand, if this user is a woman, she may feel uncomfortable or even unsafe because of the other users' reaction to her. Hence, we adopt this range measure to reflect that a single instance could lead to anxiety or trauma. We next show that the provision of any number of unisex rooms, by either converting or building extra units, will improve the safety perception.

Proposition 5. *Introducing unisex room(s) will alleviate safety concerns about public restrooms.*

The rationale behind Proposition A.6 is that users who have similar gender identities endogenously decide to use the same room. Therefore, women, trans, and gender non-conforming individuals who may fear harm when using public restrooms will feel safer as facilities adopt unisex rooms. For instance, suppose a user does not feel welcome in the gender-segregated restrooms and prefers the unisex one. If this user chooses to enter the unisex room, other users with similar gender identity types will also endogenously choose the unisex room. Thus, this user would feel more comfortable in using the unisex room, which is not directly captured in their utility.

4.2 Fluctuating Arrival Rates

It is common in practice for buildings to construct gender-segregated restrooms with the number of fixtures allocated to each of these restrooms based on the estimated gender ratio of the users. However, throughout the day, arrival rates of different genders may fluctuate. For instance, in a theatre one event may attract more users from $\mathcal{W}$ while another event may be more attractive for users from $\mathcal{M}$. We will demonstrate the benefit of unisex rooms in this environment. To this end, we propose a setting that we refer to as the *balanced* layout with the same number of total fixtures as $n_0 + n_1$, in which there are $\bar{n}_u := \max\{n_0, n_1\}$ unisex rooms, $\bar{n}_0 \geq 1$ women's room and $\bar{n}_1 := n_1 + n_0 - \bar{n}_0 - \bar{n}_u \geq 1$ men's rooms. We denote by $\hat{W}_0$, $\hat{W}_1$, and $\hat{W}_u$ the equilibrium expected wait time of the women's, the men's and the unisex rooms in the balanced layout, respectively. We next show for sufficiently low sensitivity to gender identity, the disparity of wait times is lower in the balanced layout with $\bar{n}_u$ unisex rooms than that of the status quo.

Proposition 6. *For any Λ_0 and Λ_1, there exists $\hat{t}$ such that for $0 \leq t \leq \hat{t}$, the wait-time disparity of the balanced layout is less than that of the status quo, i.e., $| \hat{W}_0 - \hat{W}_1 | \leq | w_0 - w_1 |$.*

Proposition A.7 holds for any arrival rate, and thus, implies that having a unisex restroom can improve the wait-time disparity between the gender-segregated restrooms when the arrival rates change over time. The intuition is that the unisex rooms provide more toilets to serve spikes in the bathroom needs of either $\mathcal{W}$ or $\mathcal{M}$, since these unisex rooms admit all types of users. When there is a spike in restroom demand from $\mathcal{W}$ (resp., $\mathcal{M}$), a portion of them will spill over to the unisex room, and thus, a larger portion of the users of the unisex rooms will consist of $\mathcal{W}$ (resp., $\mathcal{M}$). Such flexibility of unisex rooms in assisting any type of users help better serve fluctuating demand across types, and thus, reduce wait-time disparity, compared with the gender-segregated-only restrooms.

5 NUMERICAL STUDY

In this section, we use a numerical experiment to study a particular configuration of the public restroom, to show how a unisex restroom affects the rooms' wait times and the users' utilities. In a simulation study, [7] use a stadium from which they obtained a dataset for washroom use and investigate the appropriate ratio of women's to men's fixtures. We do not have access to such a dataset. Hence, we replicate their setting, a toilet system in one corner of the stadium, as our status quo system and use the parameters they consider in our numerical study.[12]

Following Huh et al. [7], we consider a user who enters a restroom as one of two classes: class 1 users' need is for service type 1, urination, and class 2 users' need is for service type 2, defecation, changing a sanitary pad or tampon, etc. The likelihood of a user $i \in \mathcal{W}$ belonging to class 2 is $1/21$, and the likelihood that a user $j \in \mathcal{M}$ belongs to class 2 is $1/16$. The service times for urination for users from $\mathcal{W}$ and $\mathcal{M}$ follow exponential distributions whose means are 50 and 100 seconds, respectively. Also, the service time for defecation for everyone follows the exponential distribution with the average being 420 seconds. We consider the status

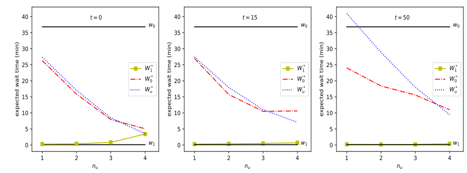
[12] All parameters related to our queueing model are drawn from Huh et al. [7] except for parameters related to the gender disutility such as t and distributions $f_0(v)$ and $f_1(v)$, $v \in [0, 1]$ on gender identity, which are absent in their study.

quo of the system as in Huh et al. [7]: the women's room comprises 3 toilet stalls that accommodate both classes of users from $\mathcal{W}$, and the men's room comprises 2 toilet stalls and 6 urinals. Class 1 users from $\mathcal{W}$ and class 2 users from $\mathcal{W}$ and $\mathcal{M}$ can only use toilets, whereas class 1 users from $\mathcal{M}$ can use both toilets and urinals. Huh et al. [7] estimate that the stadium's capacity is 1,829 seats and each game lasts 150 minutes. As they do, we simulate the restroom usage for up to 150 minutes and up to the capacity of the stadium. We simulate the dynamics for 1,500 times to estimate the expected wait time for each of the restrooms. We consider that arrival processes of users from both $\mathcal{W}$ and $\mathcal{M}$ are independent Poisson processes with arrival rates of 3.2 persons per minute. From our simulation the expected wait time for the women's room in the status quo arrangement is 36.83 minutes, while that of the men's room is 0.12 minutes. Using the measure of the probability of waiting in line for more than a minute, our simulation generates consistent results as what are reported in Huh et al. [7], i.e., that probability of waiting is 98% for the women's room and 1% for the men's room.

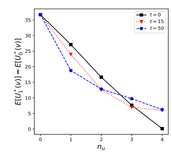
For the purpose of computing users' utility (which is not considered in Huh et al. 7), we normalize the waiting cost coefficient to 1, i.e., $c = 1$. Therefore, in our status quo, the wait-time disutility for users from $\mathcal{W}$ from using the women's room is 36.86, while the wait-time disutility for users from $\mathcal{M}$ from using the men's room is 0.12. We then consider three parameters for sensitivity to gender identity, $t = 0$, 15, and 50, to account for the populations whose sensitivity to the notion of gender identity is negligible, moderate, or strong, respectively. The rationale for these parameters is that users with extreme types 0 and 1 will favor standing in line for up to t more minutes to use a restroom with signage aligned with their gender identity. Therefore, the scales of the parameters make the wait time and gender identity disutilities comparable. Moreover, we consider the same type distribution function for $\mathcal{W}$ and $\mathcal{M}$: $f_l(v) = (8-10v)/4.875$ for $0 \leq v < 0.5$, and $(0.5+5v)/4.875$ for $0.5 \leq v \leq 1$, $l = 0, 1$ (see its illustration in Figure B.1 of Online Appendix B). The rationale for considering such a function is that the masses of users at the two extreme points of the Hotelling lines are denser than those in the middle. Also, the number of people who prefer the gender-segregated rooms is larger than the number of people who prefer the unisex ones.

We first study the impact of converting men's fixtures to unisex and present the three rooms' equilibrium expected wait times in Figure 5(a). This figure shows that as we convert men's fixtures to unisex, the expected wait times for the women's and the unisex restrooms drop dramatically, while the expected wait time for the men's restroom increases slightly. For instance, for $t = 15$, one men's fixture converted to unisex results in a drop of 26% in the expected wait-time disparity between the gender-segregated rooms, and a 25% fall in the maximum utility disparity among users from $\mathcal{W}$. Moreover, we observe a significant drop of 35% in the gap between the *expected* utilities of users from $\mathcal{W}$ and the users from $\mathcal{M}$; see Figure 5(b). Such a single-unit conversion leads to an improvement of 12.6 in the total welfare for people from $\mathcal{W}$, without hurting much the utilities of users from $\mathcal{M}$: their total welfare only drops by 0.17.[13]

Figure 3: The Impact of Converting Men's Restrooms to Unisex



(a) wait time of restrooms for $t = 0$, 15, 50



(b) utility disparity of $\mathcal{W}$ and $\mathcal{M}$

Moreover, in Propositions A.2 and A.3, we show theoretically the benefits of adding a unisex restroom to the status quo, either as part of an unobservable system, or as part of an observable system with the women's restroom, respectively. In our numerical study, we observe that the impact of *adding* this extra unit on the wait times for the rooms is small compared with *converting* one of the men's fixtures into a unisex room (see Figure B.2 in Online Appendix B). This is because the status quo is such that the women's restroom is very crowded, and thus, when we introduce a unisex room, whether it be a converted or a new (added) one, a large portion of the users will spill over to use this room, leading to a crowded unisex room. Since many users from $\mathcal{W}$ will occupy the unisex room, there is not much of a difference in how we provide it.

Moreover, we consider an all-unisex restroom design with 11 toilet stalls, and we observe that the wait time for this restroom is 1.98 minutes. We observe that for $t = 15$, the total social welfare produced by the all-unisex restroom is 32.2 higher than that of the restrooms in the status quo, while for $t = 50$, it is 5.72 lower than the total social welfare of the status quo. In addition, we consider an all-unisex restroom design with 10 toilet stalls and a separate urinal in an unobservable layout and assume that the mean service time for class 1 $\mathcal{M}$ users using the urinal is 35 seconds. As a result, the wait time becomes 1.8 minutes for the all-unisex restroom and 1.4 for the urinal, which suggests a Pareto improvement in terms of the wait times, consistent with Proposition A.5.

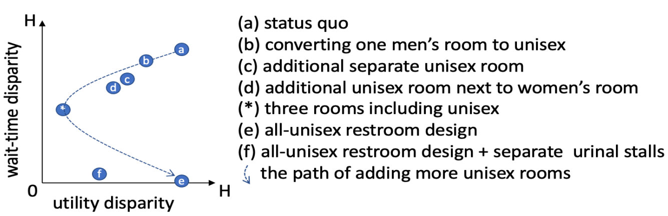
6 SUMMARY AND DISCUSSION

We summarize various designs (a)-(f) in Figure 4; different designs score differently along the two fairness dimensions, i.e., wait time and utility disparities. Point (a) reflects the status quo layout, and

[13] As the baseline total utilities for the status quo is sensitive to the specification of the reward r, we do not report those baseline numbers.

adding one unisex restroom by *converting* a unit of the men's room moves the fairness measures to point (b) (see Propositions 1 and 1). If the unisex room is an *additional unobservable* unit as shown in (c)—i.e., if the entrance is not immediately obvious to those seeking the women's restroom—the potty parity measures will improve further (see Proposition A.2). On top of that, if we can locate the additional unisex room and the women's room close together and form an *observable* system, corresponding to (d), the fairness measures will improve even further (see Proposition A.3). We can convert *more than one* unit of the men's room to unisex, enhancing the fairness measures further, leading to point (*) (see Proposition 2). However, suppose we convert sufficiently many units of the men's room: the utility disparity will start growing as we convert more units to unisex. At the extreme point (e), corresponding to an all-unisex restroom, the wait-time disparity will be zero (see Proposition 4). In contrast, the utility disparity will return to what it used to be with design (a), except that the privilege of the highest utility goes to those users who have the strongest preference for unisex restrooms. We next show that we can push the envelope by leveraging the urinals as a more efficient service for some users. For instance, at point (f), we consider a layout with an all-unisex restroom but with some urinal stalls in a separate (unobservable) place, which is shown to be a Pareto improvement on the all-unisex-only restroom design corresponding to point (e) (see Proposition A.5).

Figure 4: Summary of the Results



The implications of our work are not limited to those states and countries that are concerned about women's and LGBTQ+ people's rights. We demonstrate the benefits of the unisex rooms to a society from different perspectives, and our results are sufficiently general to apply in cases where sensitivity to gender identity may be negligible. In those cases, the main concern is efficiency that can be measured by wait times. In addition, there are countries like Ethiopia and India that lack access to public restrooms in many areas. Our results on the all-unisex restroom imply that for countries in which sensitivity to gender identity is negligible and users tend to be more comfortable using gender-segregated rooms, the benefits of adopting the all-unisex design outweigh those of gender-segregated restrooms in terms of both fairness and efficiency. This suggests that such countries could adopt unisex restrooms as much as possible both to expand the equitable access of their citizens to public restrooms and to make their public toilet system more efficient.

Lastly, we study the benefits of unisex rooms in two extensions. First, those in favour of gender-segregated public restrooms have emphasized safety concerns and fear of harm for women in public restrooms, see, e.g., [6]. On the other hand, media reports indicate gender-segregated restrooms are anxiety-provoking environments for trans and gender non-conforming individuals.[14] We show that unisex restrooms will allow users with similar identities to choose the same restroom, reducing safety concerns. Moreover, we consider situations in which the arrival rates of users fluctuate at different hours and on different days. When there is a surge of users from the women's side, the optimal allocation of fixtures requires more women's rooms. In contrast, when there is a surge of users from the men's side, more men's rooms are needed. Therefore, facilities might need dynamic control of restrooms that are gender segregated in order to improve the users' experiences. We compare a gender-segregated-restroom-only layout versus a layout with the same number of fixtures where a portion of gender-segregated restrooms from both sides have been converted to unisex ones. We show that unisex rooms naturally respond to the fluctuating arrival rates because users can easily spill over from the crowded gender-segregated restrooms to the unisex ones. Thus, unisex rooms can be counted as flexible supply for both the original users of the women's and the men's restrooms.

REFERENCES

[1] Gad Allon and Eran Hanany. 2012. Cutting in line: Social norms in queues. *Management Sci.* 58, 3 (2012), 493–506.
[2] Achal Bassamboo, Ramandeep S Randhawa, and Jan A Van Mieghem. 2012. A little flexibility is all you need: On the asymptotic value of flexible capacity in parallel queuing systems. *Oper. Res.* 60, 6 (2012), 1423–1435.
[3] Gérard P Cachon, Karan Girotra, and Serguei Netessine. 2020. Interesting, important, and impactful operations management. *Manufacturing Service Oper. Management* 22, 1 (2020), 214–222.
[4] Shiliang Cui, Zhongbin Wang, and Luyi Yang. 2020. The economics of line-sitting. *Management Sci.* 66, 1 (2020), 227–242.
[5] Suri Gurumurthi and Saif Benjaafar. 2004. Modeling and analysis of flexible queueing systems. *Nav. Res. Log.* 51, 5 (2004), 755–782.
[6] Amira Hasenbush, Andrew R Flores, and Jody L Herman. 2019. Gender identity nondiscrimination laws in public accommodations: A review of evidence regarding safety and privacy in public restrooms, locker rooms, and changing rooms. *Sexuality Research and Social Policy* 16, 1 (2019), 70–83.
[7] Woonghee Tim Huh, Jaywon Lee, Heesang Park, and Kun Soo Park. 2019. The potty parity problem: Towards gender equality at restrooms in business facilities. *Socio-Economic Planning Sci.* 68 (2019), 100666.
[8] William C Jordan and Stephen C Graves. 1995. Principles on the benefits of manufacturing process flexibility. *Management Sci.* 41, 4 (1995), 577–594.
[9] Vasiliki Kostami, Dimitris Kostamis, and Serhan Ziya. 2017. Pricing and capacity allocation for shared services. *Manufacturing Service Oper. Management* 19, 2 (2017), 230–245.
[10] Myeshia Price-Feeney, Amy E Green, and Samuel H Dorison. 2021. Impact of bathroom discrimination on mental health among transgender and nonbinary youth. *J. Adolescent Health* 68, 6 (2021), 1142–1147.
[11] Nur Sunar, Yichen Tu, and Serhan Ziya. 2021. Pooled vs. Dedicated Queues when Customers Are Delay-Sensitive. *Management Sci.* 67, 6 (2021), 3321–3984.
[12] Yanli Tang, Pengfei Guo, Christopher S Tang, and Yulan Wang. 2021. Gender-Related Operational Issues Arising from On-Demand Ride-Hailing Platforms: Safety Concerns and System Configuration. *Prod. Oper. Manag.* 30, 10 (2021), 3481–3496.
[13] Xiaoya Xu, Zhaotong Lian, Xin Li, and Pengfei Guo. 2016. A Hotelling queue model with probabilistic service. *Oper. Res. Let.* 44, 5 (2016), 592–597.
[14] Luyi Yang, Laurens G Debo, and Varun Gupta. 2017. Trading time in a congested environment. *Management Sci.* 63, 7 (2017), 2377–2395.
[15] Luyi Yang, Laurens G Debo, and Varun Gupta. 2019. Search among queues under quality differentiation. *Management Sci.* 65, 8 (2019), 3605–3623.

[14]https://www.cnn.com/2019/05/06/health/trans-teens-bathroom-policies-sexual-assault-study/index.html